Fermilab

STOCHASTIC COOLING IN THE DEBUNCHER

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STOCHASTIC COOLING IN THE DEBUNCHER:

Calculations revisited, possibilities for improvement,
and procedures for operation and measurements

I. Signal to Noise

The main difficulty in calculating the expected
performance of the Debuncher cooling system is in knowing
the signal to noise ratio (and the kicker sensitivity). The
system, as built, is clearly worse than the design report
assumed. The main difference is that signal losses in
combining and splitting signals were assumed to be smaller
than is actually the case. The following tables summarize my
current understanding of the parameters which affect the
signal to noise ratio and kicker sensitivities.

Table I. Pickup Sensitivity

Pickup impedance (Z_o)	82.7	Ohm
Pickup sensitivity (d)	1.66	

The expected signal is given by:

$$P_s = \frac{1}{2} e^2 f_o Z_o \frac{d^2}{h^2} \beta_p \epsilon_o W$$

However, there are some losses in combining signals as
given by Tables II and III for pickups and kickers
respectively.

Table II. Pickup losses

Item	2 GHz	4 GHz	
Primary Combiner Loss	.15	.15	dB
Cable	.10	.20	dB
Second combiner	.45	.70	dB
Cable	.10	.20	dB
Feedthru	.10	.10	dB
Cable	.10	.10	dB
Hybrid (combiner)	.15	.35	dB
Coupler	.15	.15	dB
Cable	.10	.20	dB
Total	1.4	2.2	dB

Table III. Kicker losses

Item	2 GHz	4 GHz	
Primary combiner loss	.15	.15	dB
Cable	.10	.20	dB
Combiner network	1.5	2.5	dB
Total	1.8	2.8	dB

The noise temperatures are fairly well known although there is some doubt about the exact temperature of the resistors in the array and whether their noise is purely thermal in nature. Approximate values (probably good to 20%) are given in table IV.

Table IV. Noise Temperatures

Resistor noise temperature	100	deg K
Amplifier noise temperature	80	deg K

It is worth asking how well these expectations agree with measurements. The beam measurements at Argonne, which included losses in the primary combiner, indicated sensitivities 20% lower than calculated (pbar note 379). I feel the Argonne measurement is in agreement with calculations, but I have heard it stated by others that the measurement is good to 5 or 10%.

We also have some TEV I data which measures signal to noise. One measurement is summarized in Table IV.

Table IV. Debuncher Signal to Noise

Pickup	2.0GHz	2.4GHz	2.8GHz	3.2GHz	3.6GHz	4.0GHz	
H1	-14	-13	-13	-14	-13	-16	dB
H2	-13	-11	-11	-11	-12	-15	dB
H3	-14	-11	-11	-13	-12	-14	dB
H4*	-17	-15	-14	-15	-17	---	dB
V1	-10	-8	-7	-8	-8	-12	dB
V2	-9	-8	-9	-10	-10	-10	dB
V3	-8	-7	-7	-9	-9	-10	dB
V4	-12	-9	-9	-9	-9	-11	dB

*The H4 cryogenic amplifier was known to be sick when the data was taken.

There are a number of things that were wrong with this measurement:

1. There was no check to see how much of the signal was in the common mode.

2. There was no check on what the initial emittance was – or whether it was changing – in time the measurements were made (the vertical measurements were done second).

3. There was no measurement of all the pickups in a system summed together.

Nonetheless, it is possible to compare this measurement with the expectations above provided that we assume no common mode signal and a uniform distribution in emittance. The signal to noise ratio is given by:

$$\frac{S}{N} = \frac{P_s}{P_n} \qquad \text{where} \qquad P_n = k(T_r+T_a)W$$

$$= \begin{array}{l} -11 \text{ dB @ 2 GHz} \\ -13 \text{ dB @ 4 GHz} \end{array} \qquad \text{at midband}$$

where the number of electrodes (n_p) is 32 and the average emittance (ϵ_o) is assumed to be 5π mm-mrad (corresponding to a full aperture of 12π mm-mrad). The agreement appears to be adequate given the poor quality of the data.

One other effect reduces the sensitivity of the pickup and kicker arrays: the relative betatron phase advance between pickups is not the ideal 0 or 180 degrees, and, as a

result, some signal is lost. The sensitivity is 94% for the horizontal and 92% for the vertical.

So far the discussion has implicitly assumed small betatron amplitudes. If the pickup response is non-linear, then the sensitivity will depend on the amplitude in a complicated way. To get a semi-quantitative estimate of these effects, the sensitivity formula derived by Ruggiero (pbar note 148) is used:

$$Z(x,y) = \frac{1}{\pi} \left(\tan^{-1}\left(\tan\left(\frac{\pi y}{g}\right)\tanh\left(\frac{\pi x^+}{g}\right)\right) - \tan^{-1}\left(\tan\left(\frac{\pi y}{g}\right)\tanh\left(\frac{\pi x^-}{g}\right)\right) \right)$$

where $x^+ = x + w$ and $x^- = x - w$

with w = effective plate width, g = pickup gap

The coordinates are chosen so that the pickup is supposed to be sensitive to betatron motion in y.

The time dependence of the signal is given by:

$$S(t) = \sum_n \delta(t - n t_o) \, Z(x(t), y(t))$$

In order to exhibit the signal in terms of the betatron sidebands, it is necessary to make a Fourier expansion of S(t). One way of making the expansion is shown below:

$$S(t) = \sum_n e^{i n \omega_o t} \sum_l \sum_m Z_{lm} e^{i\pi l x/a} \, e^{i\pi m y/b}$$

where

$$Z_{lm} = \frac{1}{2a} \frac{1}{2b} \int Z(x,y) \, e^{i\pi l x/a} \, e^{iy/b} \, dx \, dy$$

$$x(t) = x_B \cos(Q_x \omega_o t)$$

and

$$y(t) = y_B \cos(Q_y \omega_o t)$$

Since $\exp(i\pi l x_B \cos(Q_x \omega_o t)/a) = \sum_\sigma (i)^\sigma J_\sigma(\pi l x_B/a) \, e^{i\sigma Q_x \omega_o t}$

$$S(t) = \sum_{n,\sigma,\tau} e^{i\omega_o t(n + \sigma Q_x + \tau Q_y)} (i)^{\sigma+\tau} \sum_l \sum_m Z_{lm} J_s(\pi l x_B/a) \, J(\pi m y_B/b)$$

This shows that the observed signal spectrum includes, in general, all revolution harmonics plus or minus all harmonics of the tunes in both transverse planes. In specific cases, some combinations may be missing because of symmetry. In our case, the pickup is rather linear, and the only significant signal is the (normal) first side-band $\sigma=0$, $\tau=1$. While the amplitude in other modes may not always be negligible (10 to 20%), cooling effects are proportional to amplitude squared. The cooling term is proportional to amplitude but has both the pickup and kicker sensitivity included. In order for cooling to take place, the non-linear signal must couple to a similar non-linearity in the kicker which is similar in magnitude. In the case of the heating term, the effect is proportional to the power density in the pickup. Of course the non-linearities will not contribute to heating unless they overlap the first harmonic schottky lines (they do in our case since the tune is near 1/4).

Figure 1 shows the sensitivity (S=d/2, where d is given by the design book) of an ideal pickup as a function of betatron amplitude. For small amplitudes in x, the sensitivity increase monotonically with y until it reaches a maximum of almost 1 when the betatron motion barely touches the plate at its maximum. However, as x increases to very large values, the sensitivity increase flattens (this is due to the finite width of the plates). Note also that when the x and y betatron amplitudes are equal, the sensitivity is slightly decreasing as a function of amplitude.

The Debuncher electrodes have designed so that .8 < x_B/y_B < 1.9, i.e., they suffer from the loss of sensitivity when the betatron amplitude is large. Averaged over all pickups the effect for the largest amplitude particles is a 30% reduction in sensitivity. However, the geometry is favorable for small beams. When the beam size is small, the sensitivity is given by:

$$d = 2 \tanh\left(\frac{\pi w}{2g}\right)$$

which for our geometry is .827 for a 3 cm gap and .529 for a 6 cm gap. Since we expected to have signal to noise problems when the beam size was small, we chose the geometry which gave the best performance with small beams. The sensitivity of the kickers is similar but not identical since the kicker sensitivity is proportional to the derivative of the pickup signal.

My simulations have ignored the change in sensitivity as a function of amplitude for the following reasons:

1. To include the effect properly a 2-dimensional Fokker-Planck equation is required since the sensitivity depends on amplitudes in both x and y.

2. If one assumes that the betatron amplitudes are always equal in x and y, then the sensitivity is fairly flat out to moderate amplitudes. Since we are cooling in both planes, this assumption is true in the mean.

3. We have traditionally expected that the cooling process would be limited by the signal to noise ratio at the end of the cooling process. Because this ratio is poorer than expected, this assumption is less well justified.

CONCLUSION: The overall signal to noise ratio according to the current calculation (for 128 pickups including all losses but still using the small signal approximation) is .25 compared to the design calculations which assumed .47. The difference comes from losses in combining PU arrays, improved calculations of the pickup impedance, and improved calculations of the coherence (none of which went the "right" way!).

II.   Cooling calculations

Using the above best estimates of pickup and kicker sensitivities (without the amplitude dependence discussed above), I have recalculated what we can expect in terms of cooling for various TWT power levels. These calculations take into account the power limitation and the optimum gain. The gain is set to the optimum or as high as the power limitation will allow. The power quoted is at the output of the TWT - the losses in the splitter circuits have already been taken into account. Figure 2 shows the fraction of particles within a 7π mm-mrad emittance as a function of time and power level. Note that the graph shows the emittance in one plane only. The amount of beam less than 7π mm-mrad is the ordinate amount squared (assuming identical systems). Thus, with 1000W is possible to cool 96% (.98**2) of the beam to 7π mm-mrad in 2 sec. This meets, or nearly meets, the design criterion.

The curves show the feature that the high power systems have asymptotic values which are lower than the low power systems. This feature is caused by the loss of particles hitting the aperture in the early part of the cycle. The stochastic cooling equation consists of both a heating and a cooling term. Some particles initially go the wrong way. They would eventually be cooled if the aperture were not there. The gain may be modulated (lower) during the early stages so that the particles are not lost. Of course, one sacrifices initial cooling speed to do this. For example, gain modulation of the 2000W system can allow cooling 99% of the beam within 7π mm-mrad in 2 seconds.

If the accumulator acceptance is smaller than 7π mm-mrad plus mismatch errors or if the stack tail system is inefficient in handling particles with large betatron amplitudes, one might want to consider only the beam within 5π mm-mrad. This latter emittance is used for figure 2.

HOW TO DO BETTER:

These calculations do not really reveal anything new. The cooling rates are limited initially by the amount of gain available. Cooling for longer times (1-2 sec depending on the initial power) is limited by the signal to noise ratio. Cooling is not limited by schottky power; the amount of beam may be changed by a factor of 2 with little effect. A number of improvements are possible - at least in principle.

1. Apply more power with the existing TWT's. We should have enough power to run at 1000W of TWT power.

2. Get more TWT's. This helps in that the TWT's would not have to run at such high power levels. Furthermore, the amount of plumbing on the output circuitry can be reduced so that the insertion loss getting to the kickers would be about 1 dB less. There is a limit of 8W per resistor with the current arrays - this is 8*256 = 2048W.

3. Reduce the thermal noise. This is the best thing to do since it helps cooling during long cycles which are limited by the signal to noise ratio. There are several possibilities: cool the resistors to liquid helium (factor 2 improvement). Get lower noise figure amplifiers (may not be possible in the foreseeable future). The lower noise figure

amplifier offers a maximum factor of 2 improvement - however the back termination at the pickup could be replaced with - say - 10 ohms and the need for liquid helium would be avoided. Needless to say, none of these options is cheap.

4. Add a filter. Bob Shafer has previously pointed out that one could reduce the noise power between schottky bands with a filter. This reduces the power (a factor of 2), but leaves the signal to noise ratio in-band unchanged.

5. Increase the sensitivity of the pickups and kickers (nice trick if you can do it). Or not assuming cleverness just build more of them. The latter requires some fancy filter development to correct for the fact that the phase advances are not odd multiples of 90 degrees. I believe these filters are possible, but not trivial.

III. PHASING THE SYSTEM WITH THE NETWORK ANALYZER

The beam response to an applied signal has been known and used for a number of years to measure and diagnose forces acting on the beam. I have performed a number of calculations of the beam response using a formula given by van der Meer (CERN/PS/AA/80-4):

$$S_{12}(\omega) \; \alpha \;\; - \frac{1}{4} \sqrt{\beta_p \beta_k} \int \exp(i\omega(\frac{1}{\omega_o} - \tau_c) \; (e^{j2\pi Q\alpha}(1+\frac{j}{\tan(\pi\omega/\omega_o+\pi Q)})$$

$$- e^{-j2\pi Q\alpha}(1+\frac{j}{\tan(\pi\omega/\omega_o-\pi Q)}) \;) \; \Psi(\omega_o) \; d\omega_o$$

Heuristically, this formula represents the parallel connection of a large number of LC resonators of infinite Q and resonant frequencies corresponding to the revolution frequencies of the various particles. The integral approximates the sum over all particles (resonators). Near a narrow schottky band the expression becomes:

$$S_{12} \; \alpha \; \int \frac{\Psi(\omega_o) \; d\omega_o}{\omega_o^2 - (n+Q)^2 \omega^2}$$

Which shows more clearly the relationship between the resonator model and the beam response. The integral is not well defined when the denominator becomes zero. Analysis of the situation indicates that one should take the principal value integral and one half the pole term when the denominator vanishes. Physically the principal value portion is the reactive response of the beam – it is caused by the particles at different frequencies than the applied signal. The pole term is resistive and corresponds to energy absorbed by the beam.

Figure 4abcdef show the calculated beam response for a typical beam distribution in the Debuncher. The beam has been taken to be gaussian with a quadratic polynomial added so that the full width of the beam is finite and that the beam distribution is continuous and has a continuous first derivative. The beam is thus a gaussian with truncated tails. The electronic system is assumed to have a flat gain over the band and a phase which can be described by the delay of a single cable. The sign of the gain is chosen to yield cooling. The prominent feature of the amplitude response (figures 4ace) are two peaks: one at the lower sideband and one at the upper sideband. The phase response goes rapidly from +90 degrees (inductive in this convention) to -90 degrees (capacitive) as the resonant frequency is traversed. Note that this pattern repeats every schottky band – every 590 kHz.

Responses like those shown in figures 4abcdef are not observed when network analyzer beam phasing measurements are made. The distributions are much more like those calculated with the principal value integral only (leaving off the pole term). With perhaps some hindsight this phenomena is expected because of the nature of the measurement. The applied signal is generated by a very pure synthesized frequency source. The particles at the source frequency absorb energy until the betatron oscillations become so large that the particles hit the aperture. Thus, the measurement makes a small hole in the frequency distribution, and the pole or resistive term is zero. However, the hole is so small that the PV term is unaffected. After the removal of the signal the hole probably fills in fairly quickly. Figures 5abcdef show the beam response at 2,3, and 4 GHz with the pole term omitted. Notice that the resonance has now separated into two side peaks (these occur where the beam distribution has its largest derivative) and that there is now a discontinuous 180 degree phase jump at the resonance.

The beam response function is ideal for making open loop measurements of the stochastic cooling systems and for diagnosing errors in the amplitude and phase of the system. In figures 5abc the peak gain falls from about 6 dB at 2 GHz to 0 dB at 4 GHz. An optimized gain function would have a 6 dB rise over the band - instead of the flat gain assumed in the calculation and the amplitude response would therefore be (ideally) flat.

Electrical phase errors which change slowly over a schottky band are easy to spot by comparing the phase relationship of neighboring schottky bands. Figures 6abc show the phase response of a system with a 100 psec timing error. The amplitude response is the same as Figures 5ace, but the phase is advanced by 360*frequency*100psec degrees. In practice more complicated phase errors occur, but the principle is the same - look for deviations from the ideal 180 degree negative feedback.

Betatron phase errors are also easy to diagnose by comparing the center crossing point of the upper and lower sideband. The amplitude and phase response of a system with a 300 degree phase advance between pickup and kicker is shown in figures 7abcdef. The phase response of the band at the lower frequency (actually this is the upper sideband since the fractional tune is .75) is shifted down by 30 degrees, and the other side band is shifted up by a similar amount.

In summary, two things should be looked for:

1.   A flat gain response over the whole band.

2.   Negative (180 degree) feedback at the resonance of each sideband. If there is a phase advance error, then it will not be possible to correct the upper and lower sidebands individually with a phase function which varies slowly with frequency. In this case the phase error should be split between the upper and lower side bands (like figure 7).

It is unnecessary and undesireable to measure every schottky band in great detail. It suffices to measure a few points in every 10th band or so. In doing this, however, one must be careful that the desired information is really obtained. The width of the beam response increases linearly with frequency so that if the peak response is at say 14 kHz below the resonance at 2 GHz it will be at 28 kHz below the

resonance at 4 GHz. Figures 8ab compare the difference between measuring at 14 kHz below the upper sideband at both 2 and 4 GHz compared with correct method (14 kHz below at 2 GHz and 28 kHz below at 4 GHz). Figure 8a shows an 8 dB roll-off with frequency while figure 8b shows the correct 6 dB roll-off. Other, incorrect setups can produce errors which are larger or smaller depending on what is done.

If one sets up the amplitude measurement as described above, then there will be two phase errors. The first comes from measuring off resonance (this is required since there is zero response at resonance). The other comes from the fact that the phase response does not change in frequency the way the amplitude response does. The phase response - at least in this simple model - is given by the resonant response (purely inductive below resonance and capacitive above) plus the transit time delay of one revolution period (360 degrees per schottky band). The phase response of the two methods of measurement: constant offset vrs linearly increasing offset are compared in figure 9ab. Notice that the linearly increasing offset - which gave the correct amplitude information - gives a spurious slope (but the correct offset at d.c.). The constant offset method gives the correct slope but incorrect slope intercept. These problems (although small enough in magnitude) are eliminated by taking one point on either peak to the side of the resonance and averaging the result. The averaged phase should be exactly 180 degrees if there are no errors in betatron phase.

Appendix on signal to noise measurement procedures

Recommended procedure for signal to noise measurements:

1. Move either the beam or the pickups so that the amount of common-mode signal (revolution harmonic) is small. Carefully estimate the contamination of this signal component as a function of frequency.

2. Measure the noise level at each of the amplifiers using the spectrum analyzer with a wide (3 MHz) bandwidth. Cover the entire frequency range with a single sweep.

3. Inject several hundred microamps of beam. Heat the beam to fill the aperture and lose particles until the desired intensity is reached. The spectrum analyzer scales should be the same as in 2 above.

4. Measure the distribution of the beam with the scrapers. Since this measurement is destructive, different beams should be used to measure the horizontal signal and the vertical signal.

Note: By using the scrapers in an appropriate way one can measure the amplitude dependence of the signal.

Appendix on open loop gain measurements

Recommended procedure for network analyzer measurements:

1. Measure a schottky band at the lowest frequency and one at the highest frequency. Use this information to determine the revolution frequency, tune, and beam width (distance between side lobes of the resonant response).

2. Set up to measure on each of the 4 side lobes in every 10th or 20th band.

3. Average the phase and amplitude measurements of the two side lobes and plot the upper and lower sideband phase and amplitude as a function of frequency. One must interpret the averaging carefully. An correctly phased system will have approximately a 270 degree phase below resonance and a 90 degree phase above resonance – these will properly average to 180. However, a system with 90 degree phase below resonance and 270 degree above will also average (at least naively) to 180 degrees instead of the correct answer of 0. This problem underscores the value of having a couple complete schottky bands from step 1 to avoid confusion.

4. Diagnose the cooling system deficiencies. Cable delay changes may be made on the spot. More complicated amplitude and phase changes currently require addition of fixed gain and phase equalizers.

Debuncher PU sensitivity as a function of amplitude

12/2/85
JM

Sensitivity ($d/2$)

$X_B = 0$

$X_B = .4 \times g_{ap}$

$X_B = Y_B$

$X_B = .8 \times g_{ap}$

Betatron Amplitude
(in units of g_{ap})

Figure 1

Figure 2

Fraction of the beam within 5π mm-mrad

P = 2000 W
← P = 1000 W
← P = 250 W
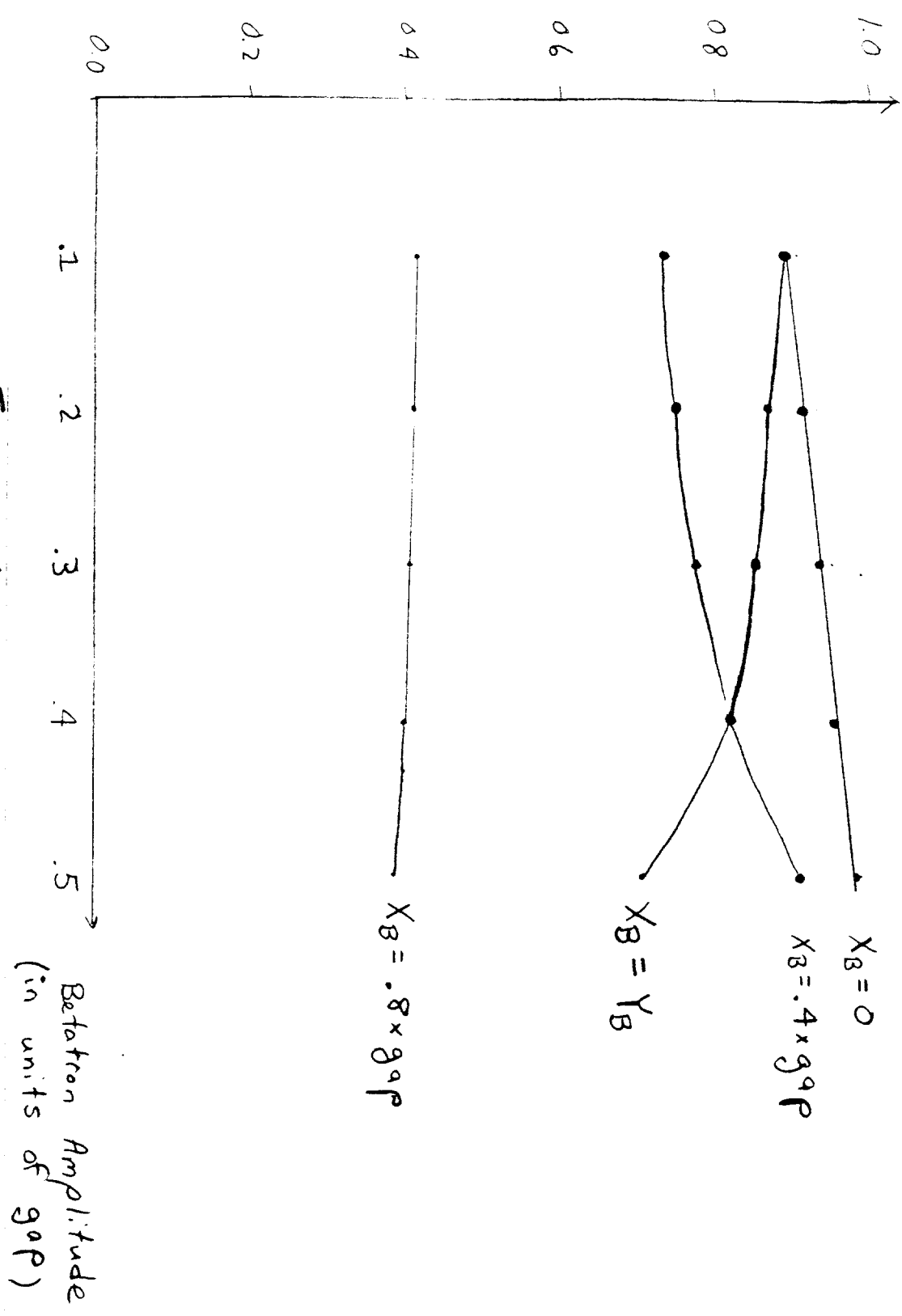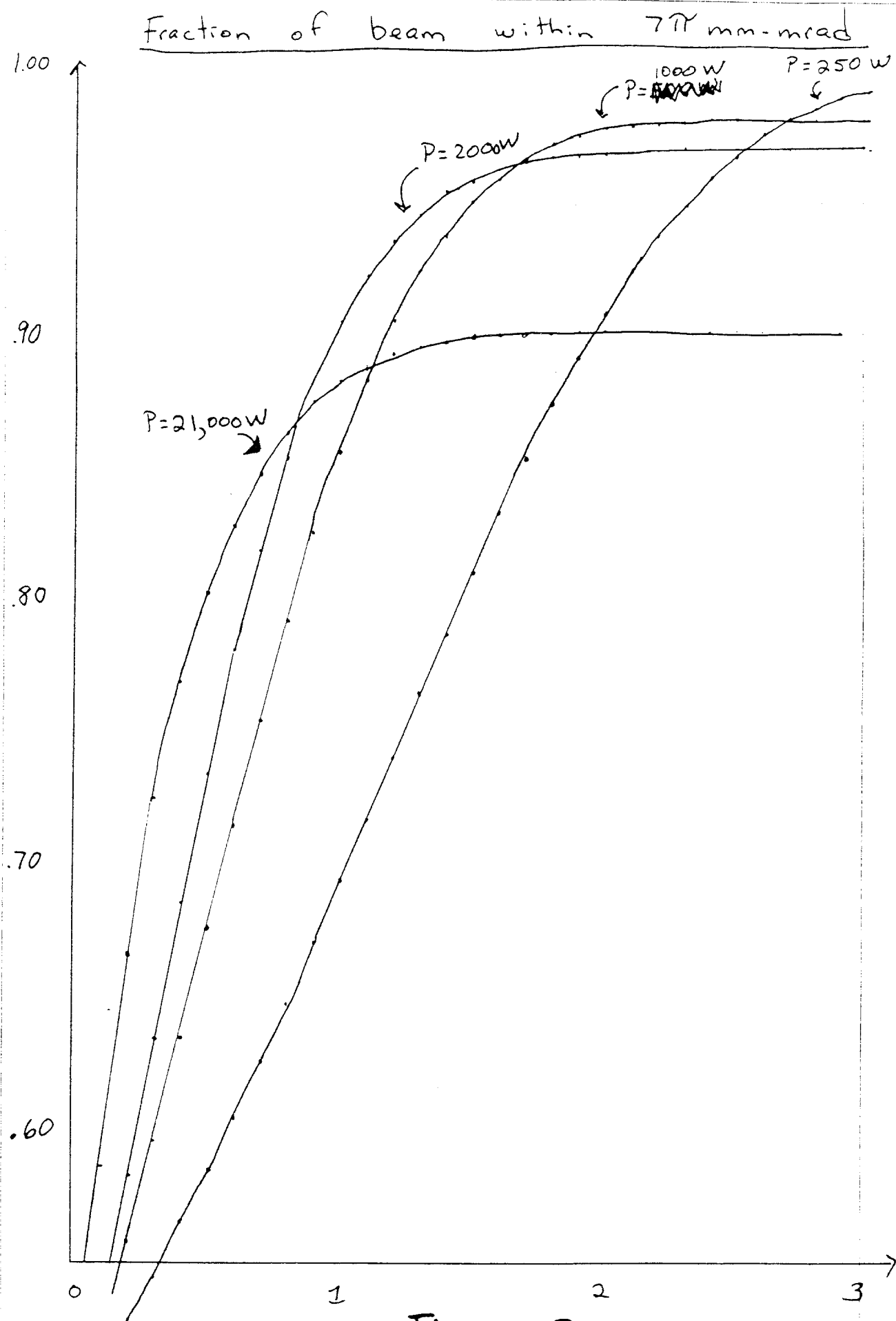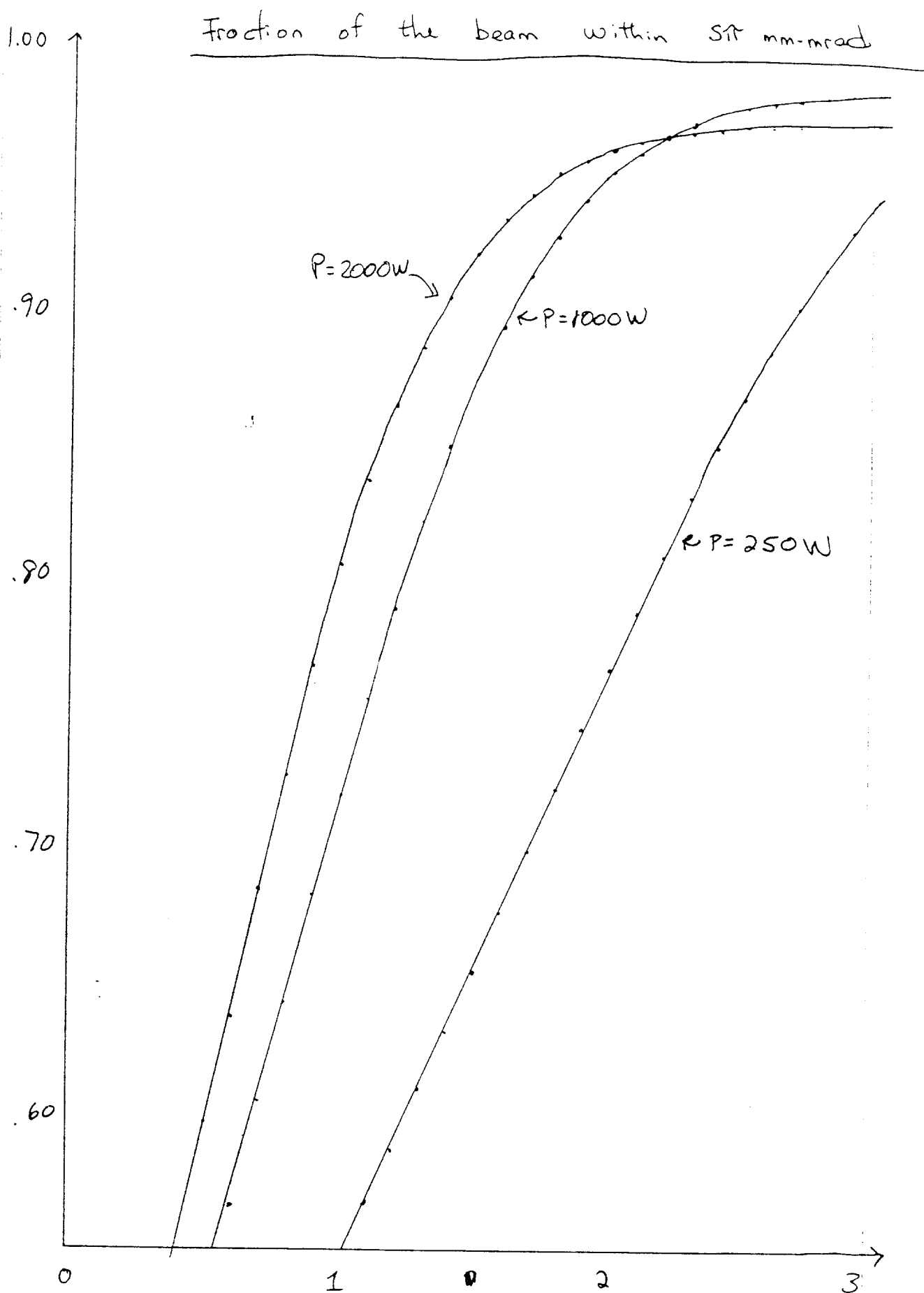
Figure 3

Band at 2GHz

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R1
                        BEAM SIGMA= .003 HZ
                        BETA PHASE ADV=270 DEG
                        CENTER FREQ=2000MHZ

Figure 4a

Band at 26Hz

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R1

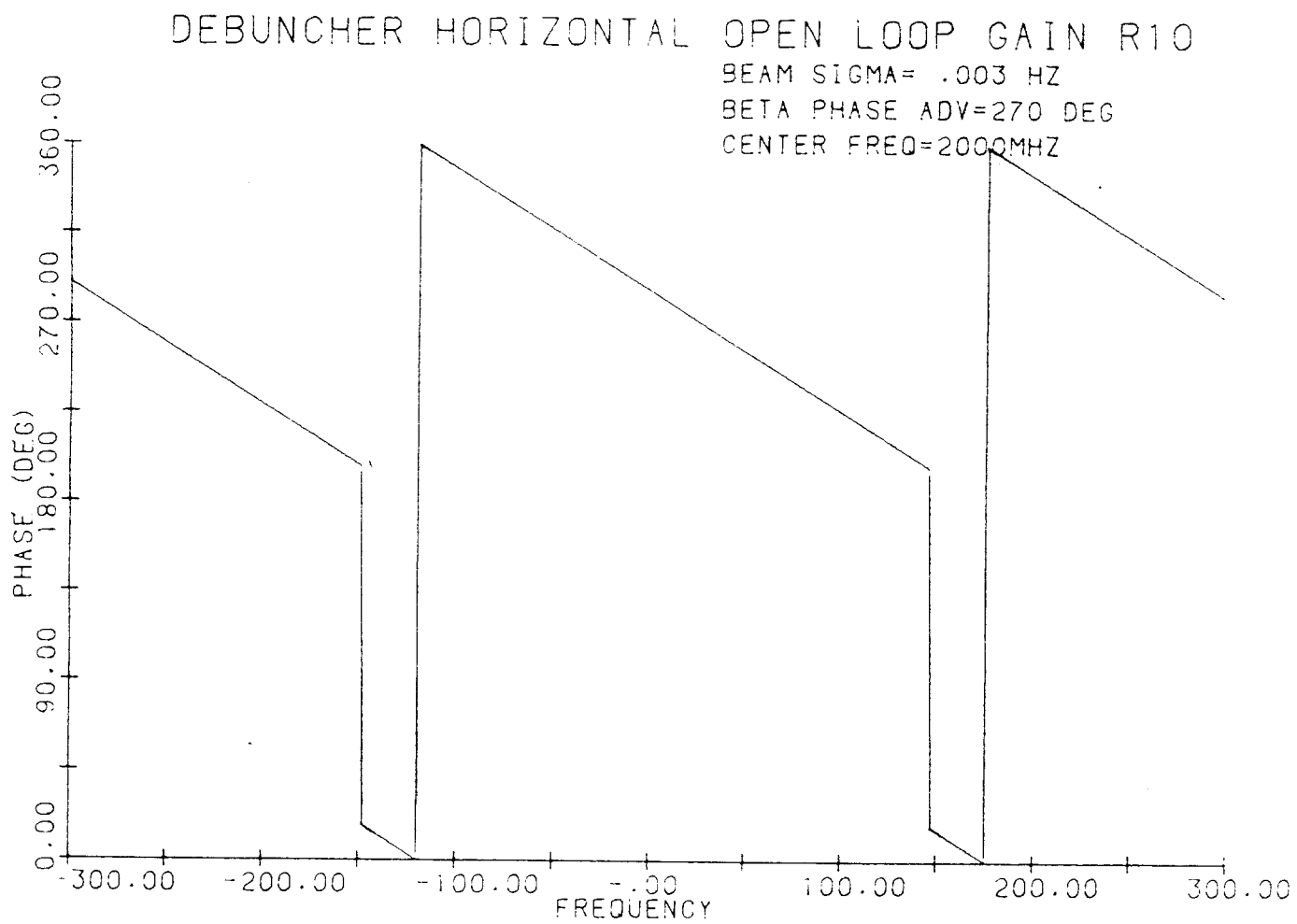BEAM SIGMA= .003 HZ
BETA PHASE ADV=270 DEG
CENTER FREQ=2000MHZ
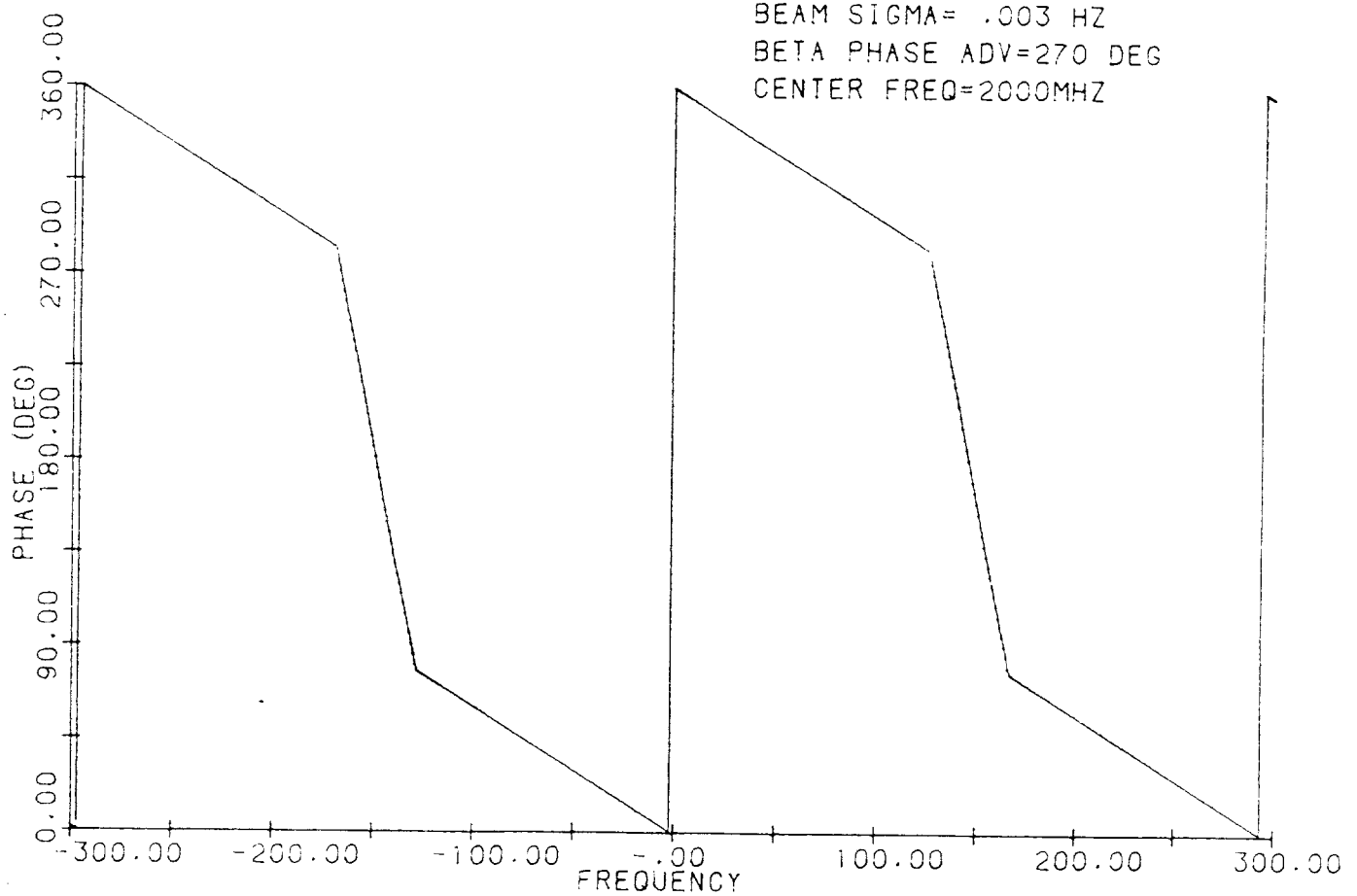
Figure 4b
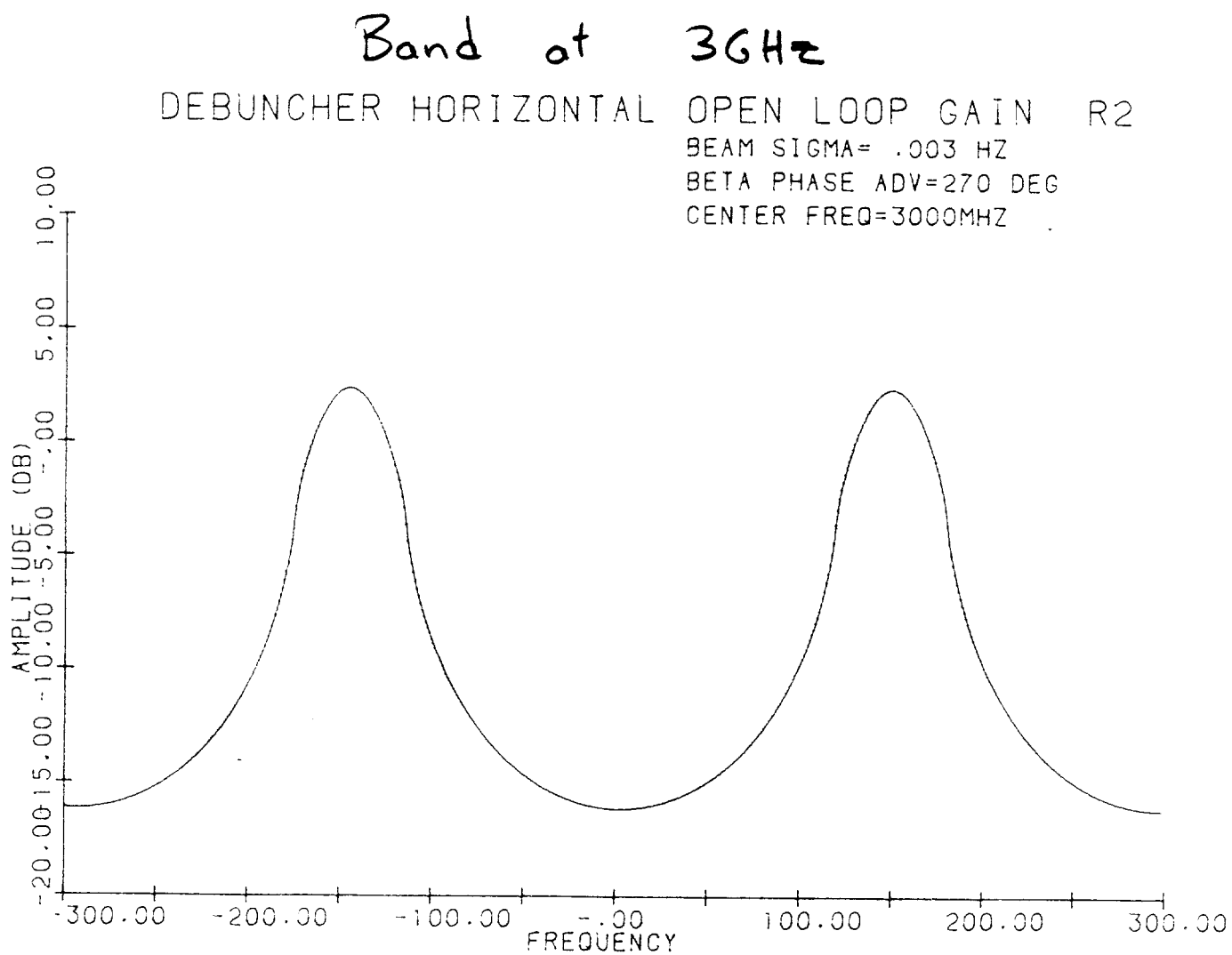
Figure 4c

Band at 3GHz

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R2

BEAM SIGMA= .003 HZ
BETA PHASE ADV=270 DEG
CENTER FREQ=3000MHZ

Figure 4d

Band at 4GHz

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R3
BEAM SIGMA= .003 HZ
BETA PHASE ADV=270 DEG
CENTER FREQ=4000MHZ

Figure 4e

Band at 46Hz

DEBUNCHER HORIZONTAL OPEN LOOP GAIN R3

BEAM SIGMA= .003 HZ
BETA PHASE ADV=270 DEG
CENTER FREQ=4000MHZ
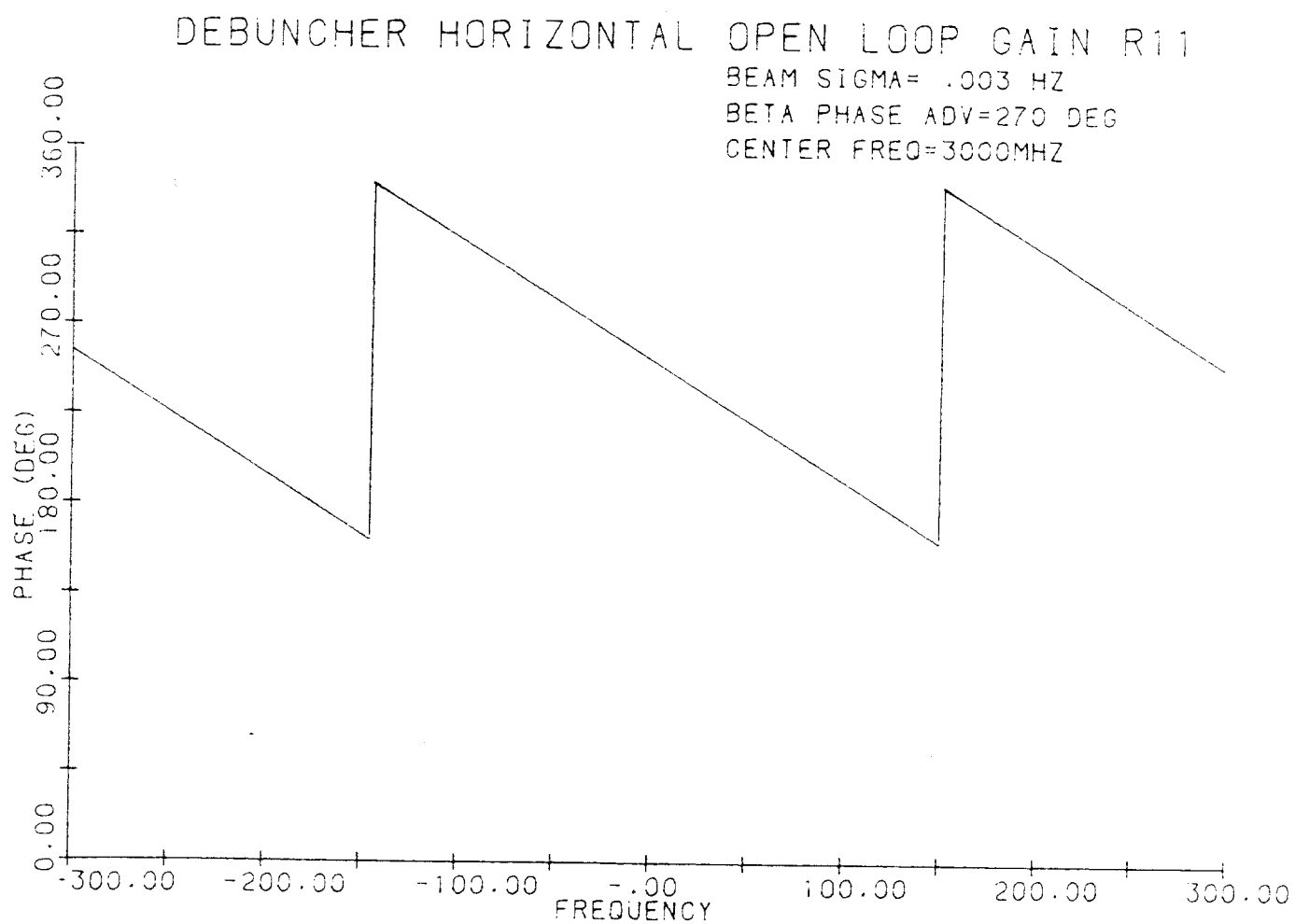

Figure 4f

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R4
BEAM SIGMA= .003 HZ
BETA PHASE ADV=270 DEG
CENTER FREQ=2000MHZ

Figure 5a

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R4
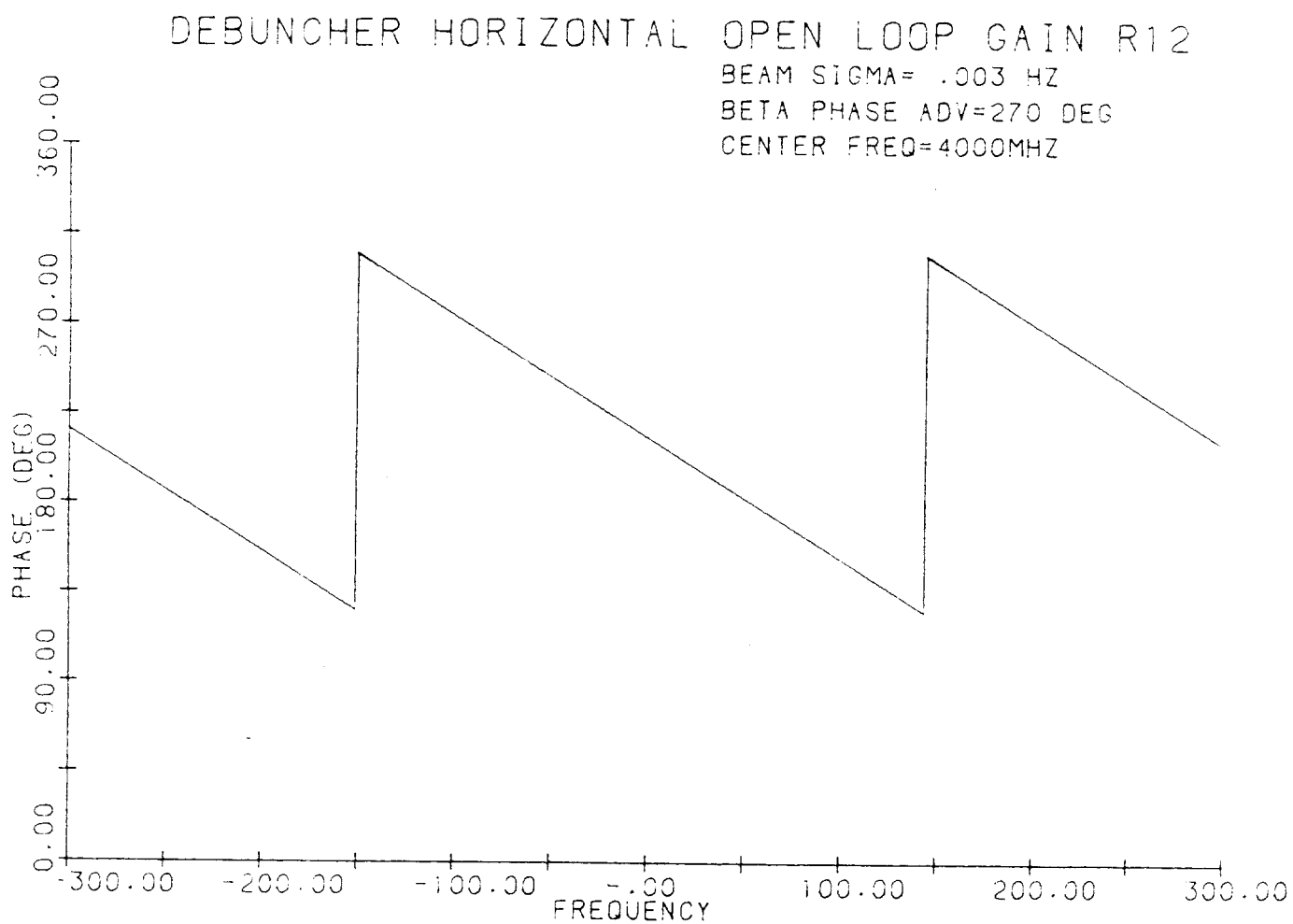
BEAM SIGMA= .003 HZ
BETA PHASE ADV=270 DEG
CENTER FREQ=2000MHZ

Figure 5b

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R5
BEAM SIGMA= .003 HZ
BETA PHASE ADV=270 DEG
CENTER FREQ=3000MHZ

Figure   5c

Figure 5d

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R6

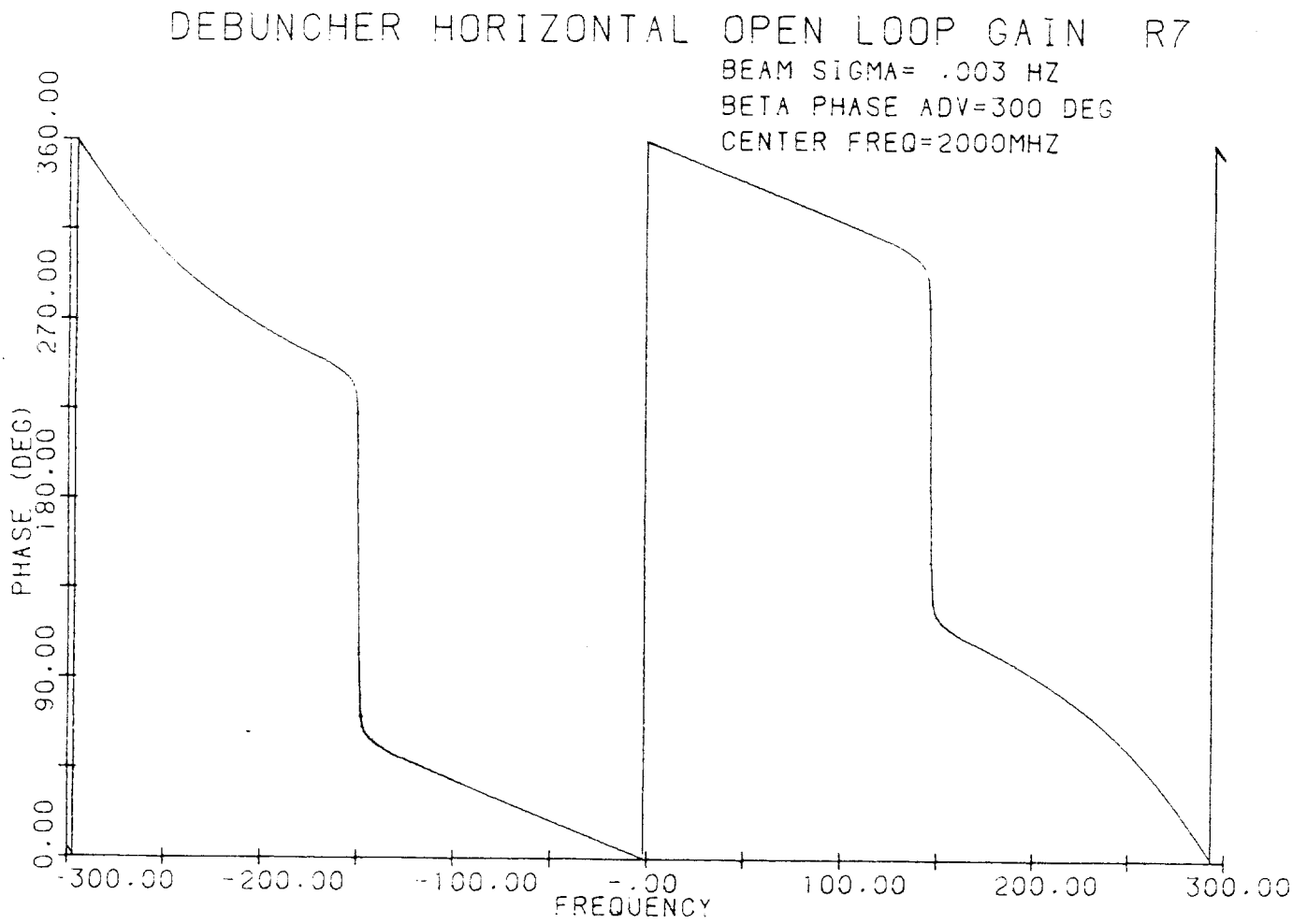BEAM SIGMA= .003 HZ
BETA PHASE ADV=270 DEG
CENTER FREQ=4000MHZ

Figure 5e

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R6
BEAM SIGMA= .003 HZ
BETA PHASE ADV=270 DEG
CENTER FREQ=4000MHZ

Figure 5f

DEBUNCHER HORIZONTAL OPEN LOOP GAIN R10
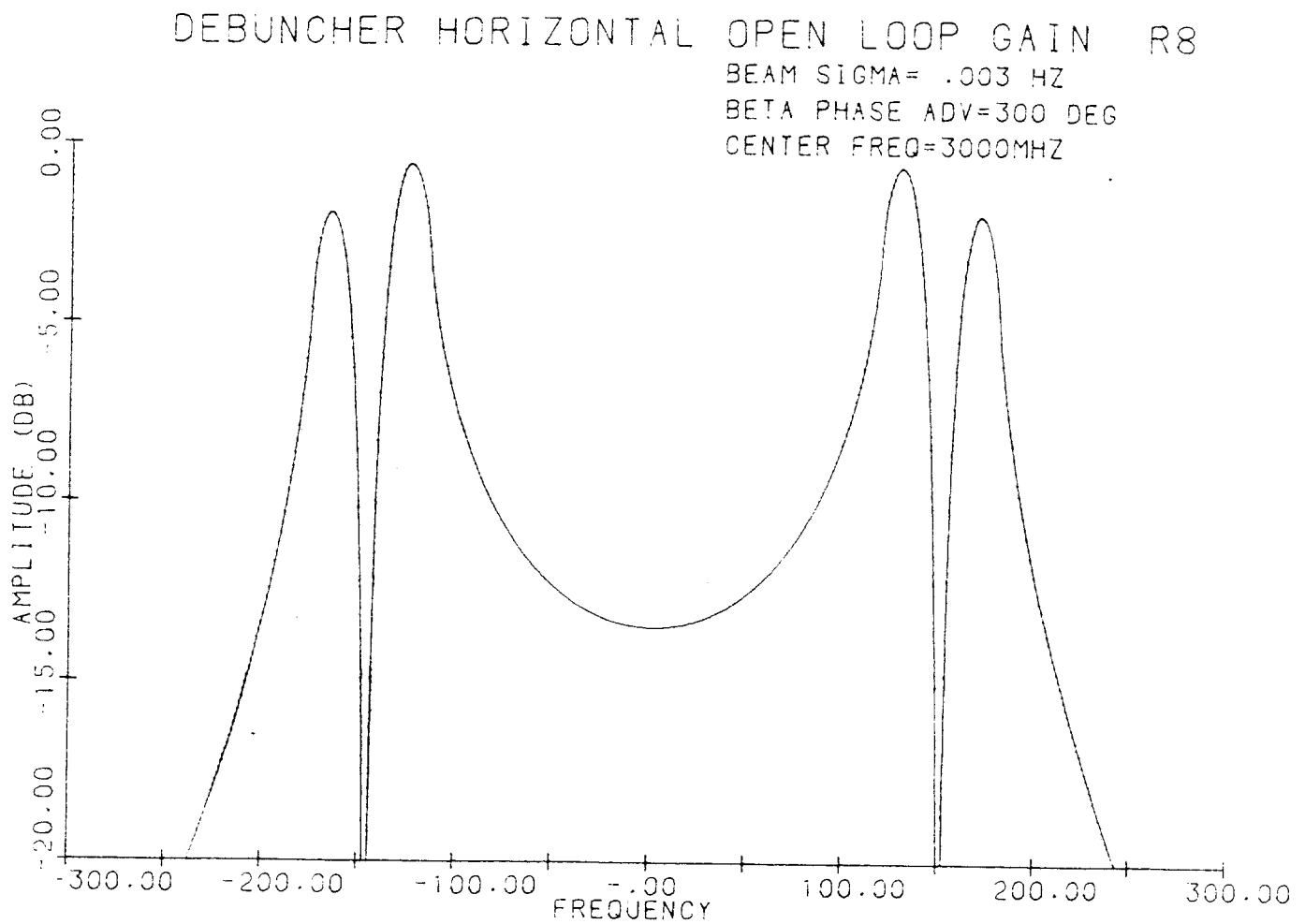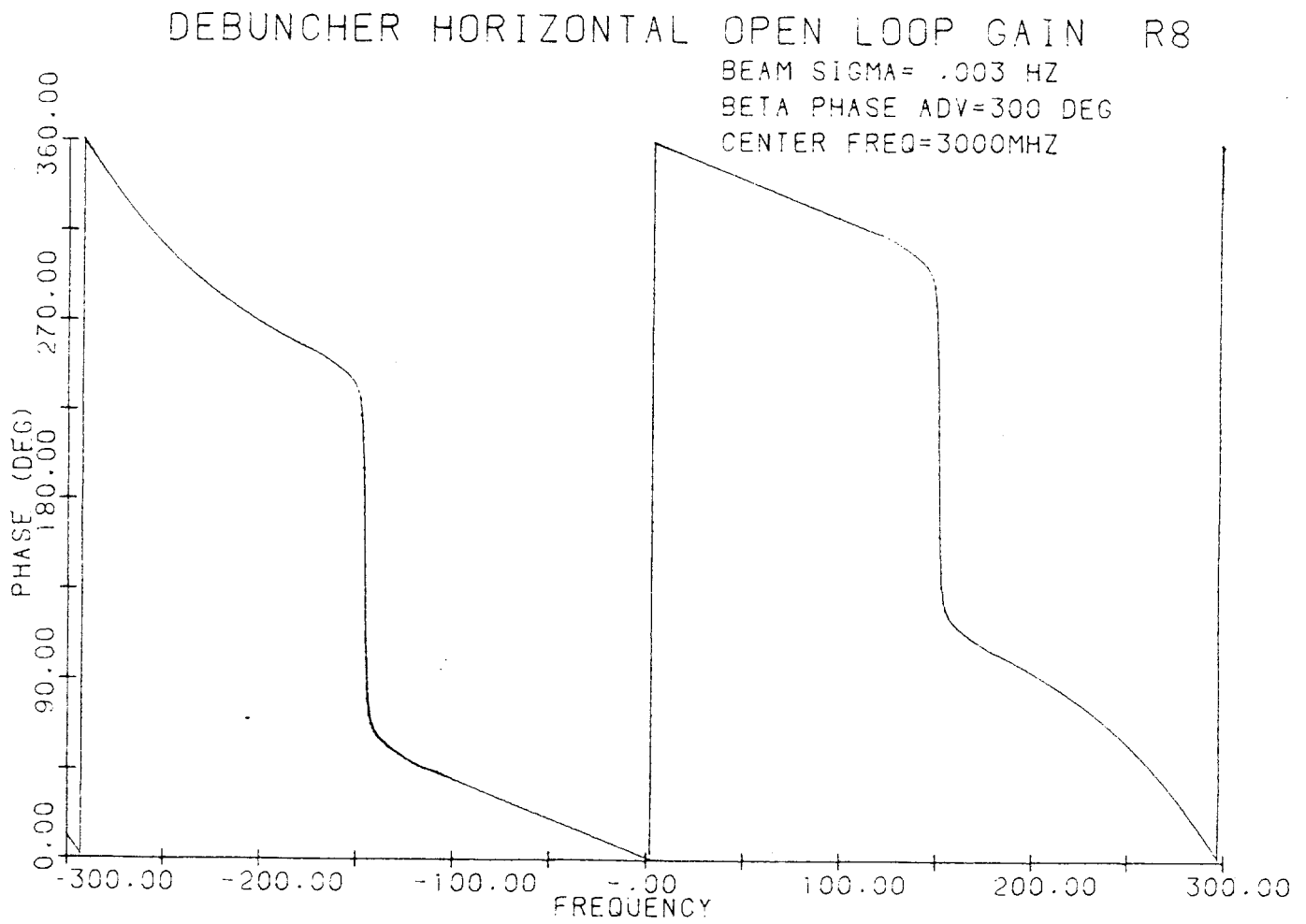BEAM SIGMA= .003 HZ
BETA PHASE ADV=270 DEG
CENTER FREQ=2000MHZ

Figure 6a

Figure 6b

DEBUNCHER HORIZONTAL OPEN LOOP GAIN R12

BEAM SIGMA= .003 HZ
BETA PHASE ADV=270 DEG
CENTER FREQ=4000MHZ

Figure 6c

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R7

BEAM SIGMA= .003 HZ
BETA PHASE ADV=300 DEG
CENTER FREQ=2000MHZ

Figure 7a

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R7
BEAM SIGMA= .003 HZ
BETA PHASE ADV=300 DEG
CENTER FREQ=2000MHZ

Figure 7b

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R8

BEAM SIGMA= .003 HZ
BETA PHASE ADV=300 DEG
CENTER FREQ=3000MHZ

Figure 7c

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R8

BEAM SIGMA= .003 HZ
BETA PHASE ADV=300 DEG
CENTER FREQ=3000MHZ
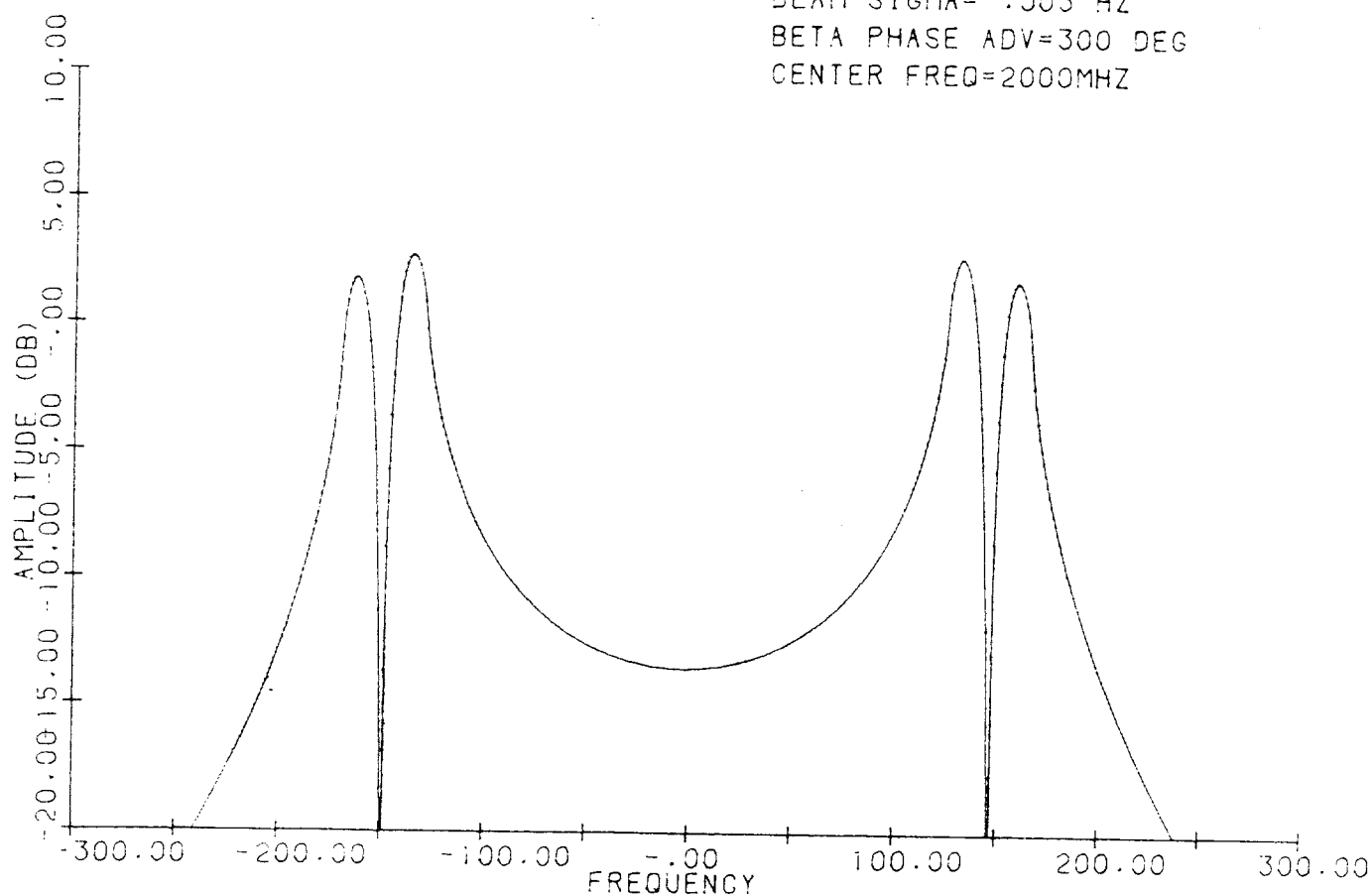
Figure 7d

DEBUNCHER HORIZONTAL OPEN LOOP GAIN R9

BEAM SIGMA= .003 HZ
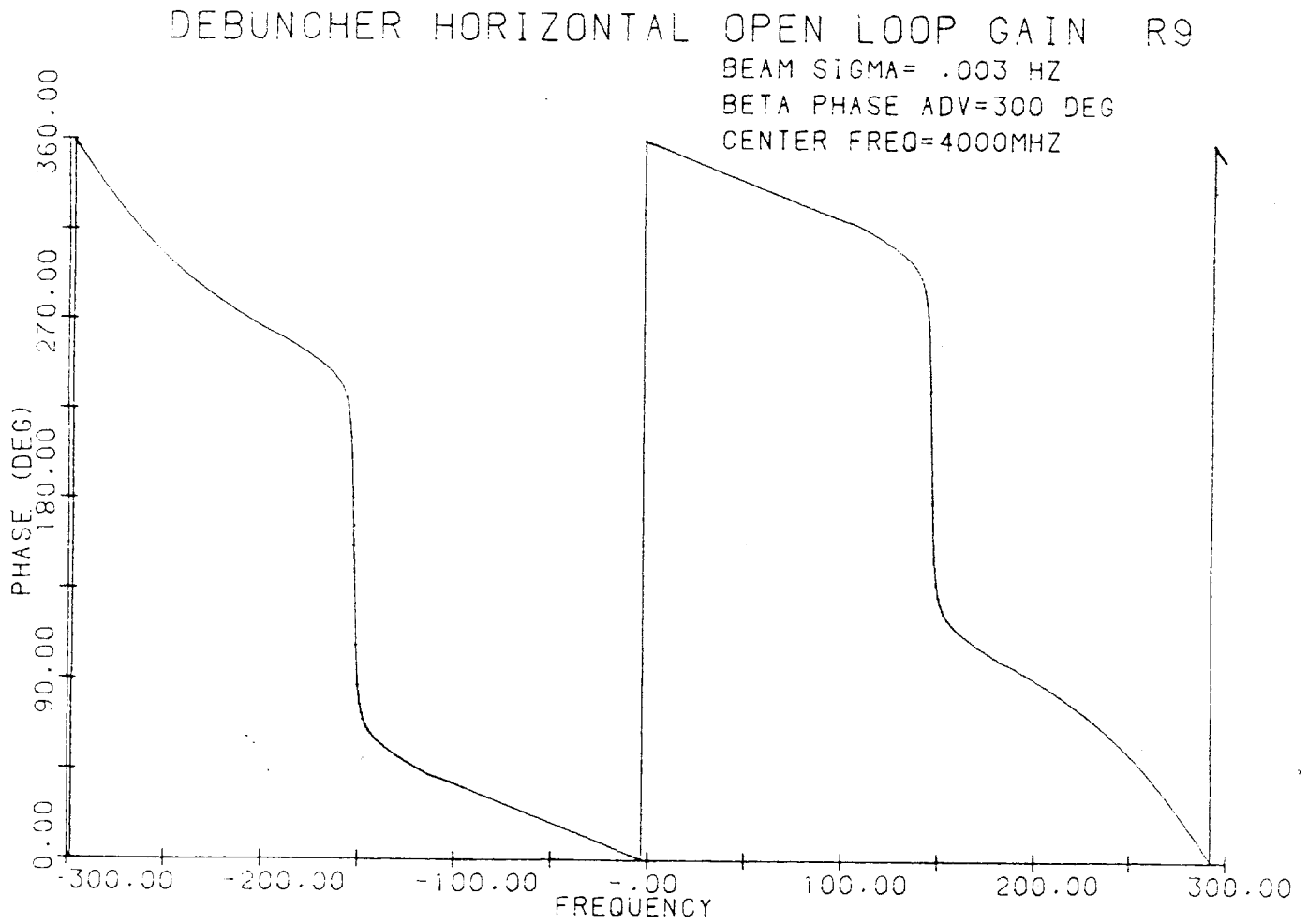BETA PHASE ADV=300 DEG
CENTER FREQ=4000MHZ

Figure 7e

DEBUNCHER HORIZONTAL OPEN LOOP GAIN   R9
BEAM SIGMA= .003 HZ
BETA PHASE ADV=300 DEG
CENTER FREQ=4000MHZ
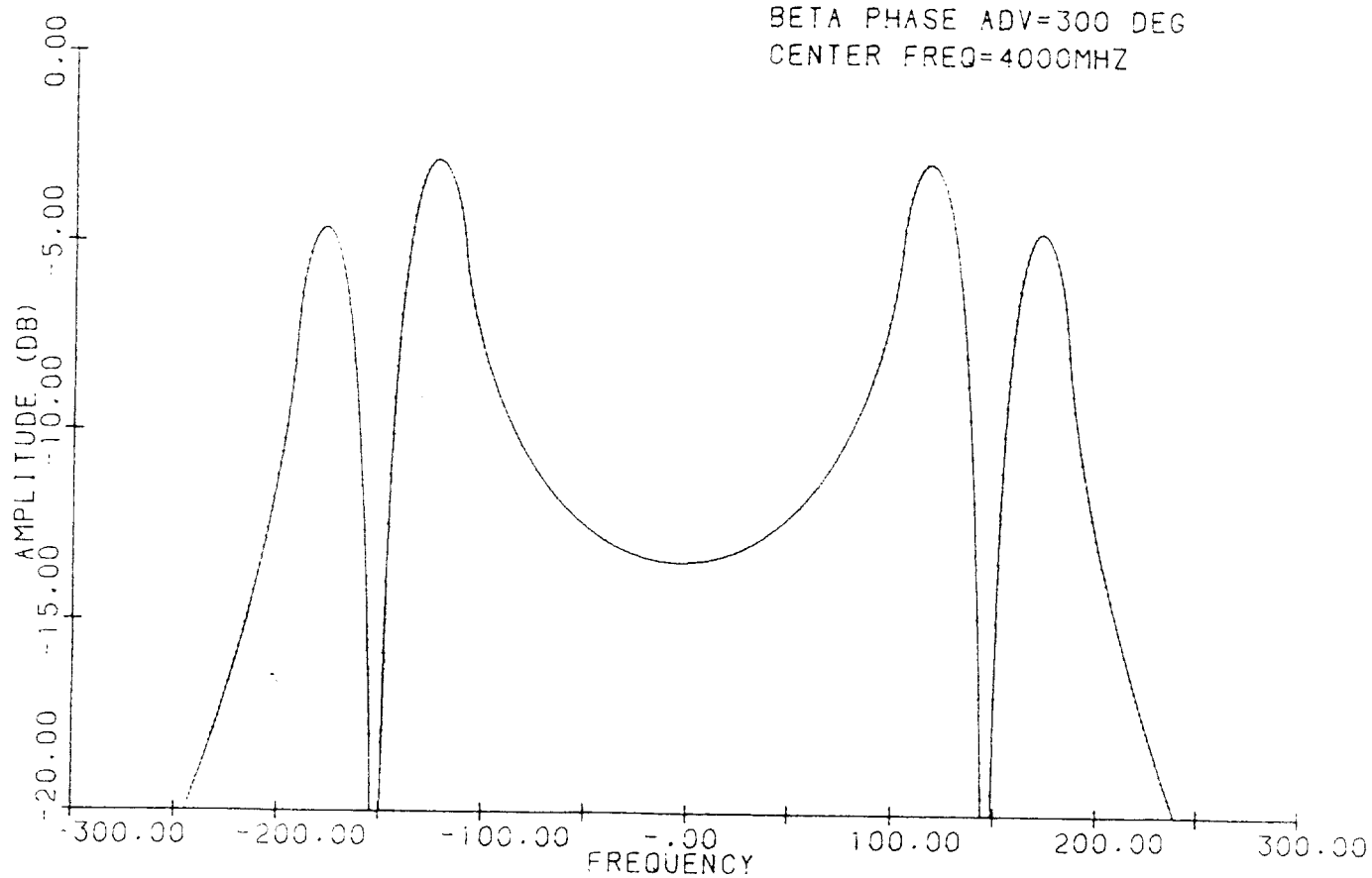
Figure 7f

DEBUNCHER HORIZONTAL OPEN LOOP GAIN R17
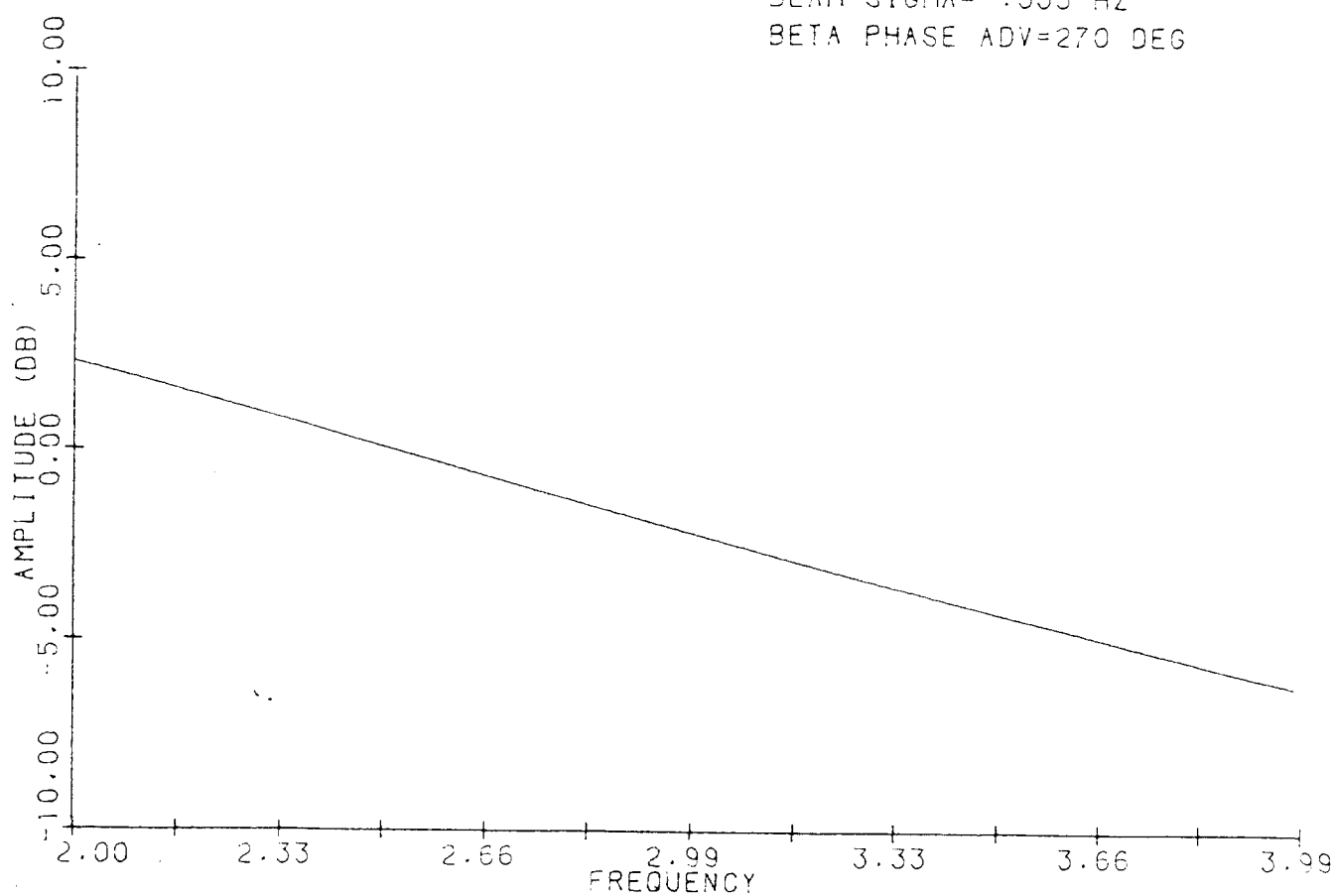
BEAM SIGMA= .003 HZ

BETA PHASE ADV=270 DEG

Figure 8a

DEBUNCHER HORIZONTAL OPEN LOOP GAIN R21

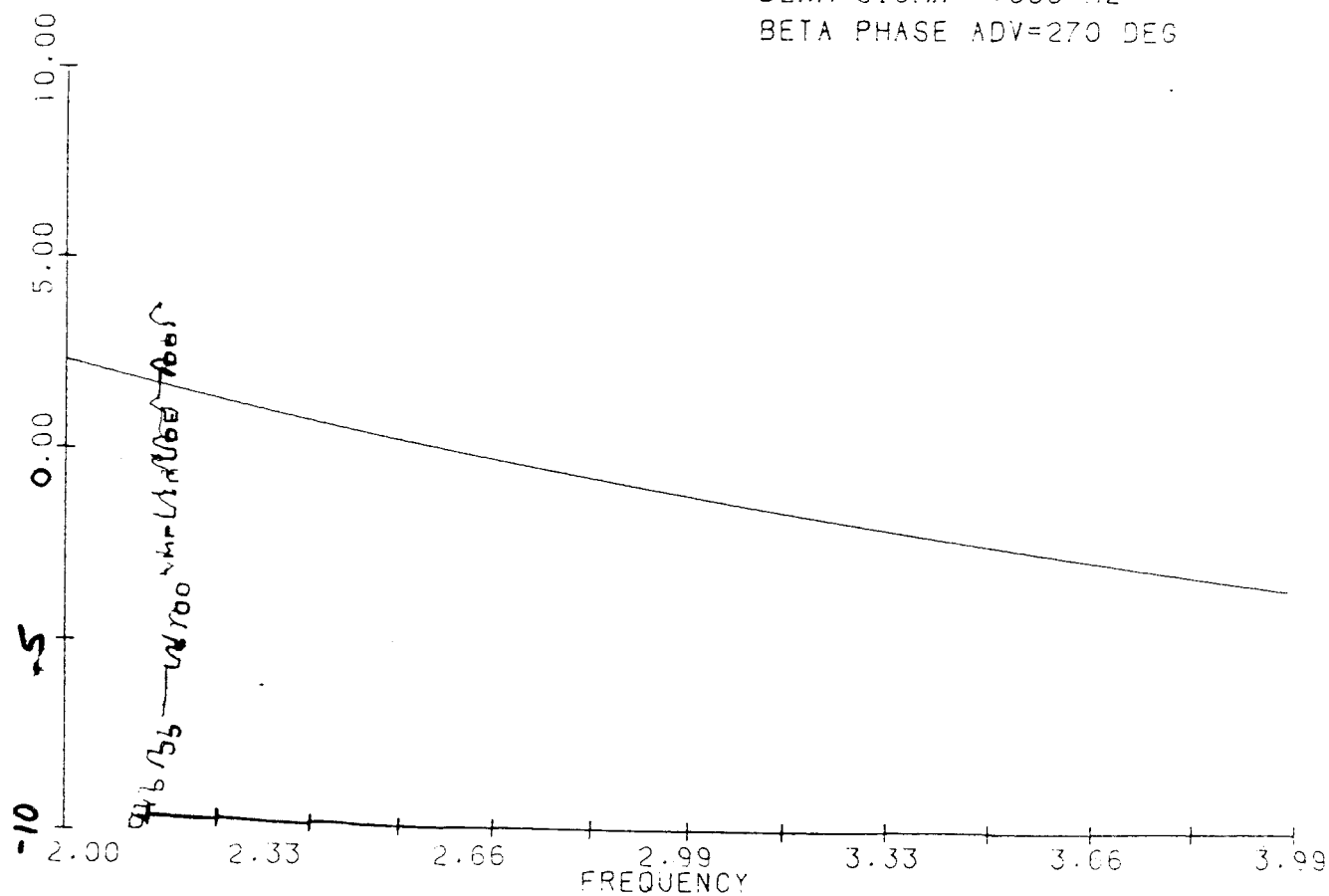BEAM SIGMA= .003 HZ

BETA PHASE ADV=270 DEG

FREQUENCY

Figure 8b